The Pythagorean theorem: For a right triangle, the square on the hypotenuse equals the sum of the squares on the other two sides
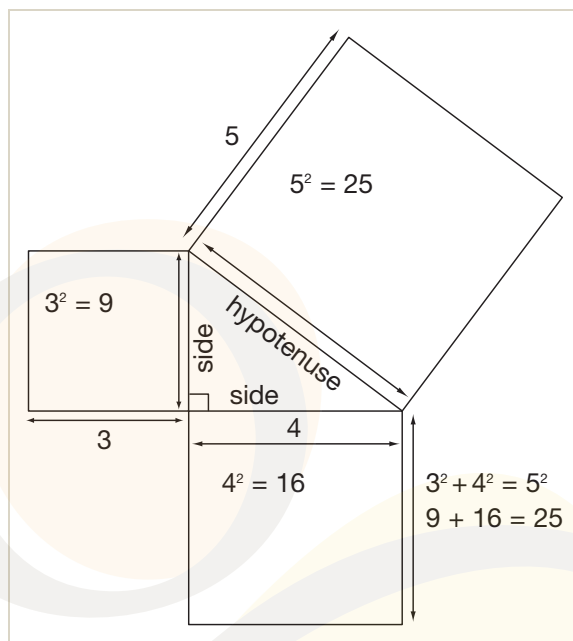
# 🏵 Pythagorean Theorem

## Background Information

Pythagoras (ca 570 BC–ca 490 BC) was a famous Greek thinker. He developed new ideas in a wide range of subjects, including music and astronomy as well as mathematics. In mid-life, Pythagoras moved to Italy and started a religious society made up of men and women. He and his followers observed dietary restrictions and tried to live pure and good lives. Pythagoras' name has been given to a theorem about right triangles. A theorem is an idea that can be proven. Historians do not know for sure that it was Pythagoras who first came up with the theorem, but the Pythagorean theorem applies to right triangles. It states that the square of the length on the hypotenuse, the longest side

of a right triangle, equals the sum of the squares of the lengths on the other two sides. For example, if a right triangle has one side that is 3 units long and another side that is 4 units long, the squares on these sides will measure 9 square units and 16 square units, respectively. The square on the hypotenuse will measure (9 + 16) square units, or 25 square units, and the hypotenuse will be 5 units long because 5 squared equals 25. The 3:4:5 example is a special case that is easy to illustrate and to understand, but the theorem holds true for any right triangle.

By Year 6, many Montessori students will be ready to understand and experience the Pythagorean theorem. They can convince themselves of its truth by manipulating equivalent squares and triangles. Then the students can work with the 3:4:5 example, counting and rearranging squares to demonstrate that the theorem is true. Finally, the students can investigate an

extension of the Pythagorean theorem that was developed by the Greek mathematician Euclid (ca 325 BC–ca 275 BC). For all three explorations, the students use the Theorem of Pythagoras material illustrated in the Introduction to the geometry section.

The Euclidean theorem about right triangles takes the Pythagorean theorem one step further. It supposes that a line is dropped from the apex of a right triangle straight down to the hypotenuse, meeting the hypotenuse at right angles and continuing on vertically through the square on the hypotenuse. It states that the two rectangles created by this dividing line will each be equivalent to one of the squares on the sides of the triangle. In other words, not only does the square on the hypotenuse equal the sum of squares on the other two sides, but the square on the hypotenuse can easily be divided into two rectangles, each equal to the square of a side.

Montessori students verify Euclid's theorem using equivalent figures. With reference to the above illustration and to the Theorem of Pythagoras material illustrated in the introduction to the geometry section of this manual, the central concepts the students consider regarding Euclid's theorem are as follows:

- Red square = blue square + yellow square (Pythagorean theorem).

- Yellow square = yellow parallelogram, and yellow parallelogram = large red rectangle. Therefore, yellow square = large red rectangle.



Euclid's theorem:
Square $a^2$ is equivalent to rectangle xc
Square $b^2$ is equivalent to rectangle yc
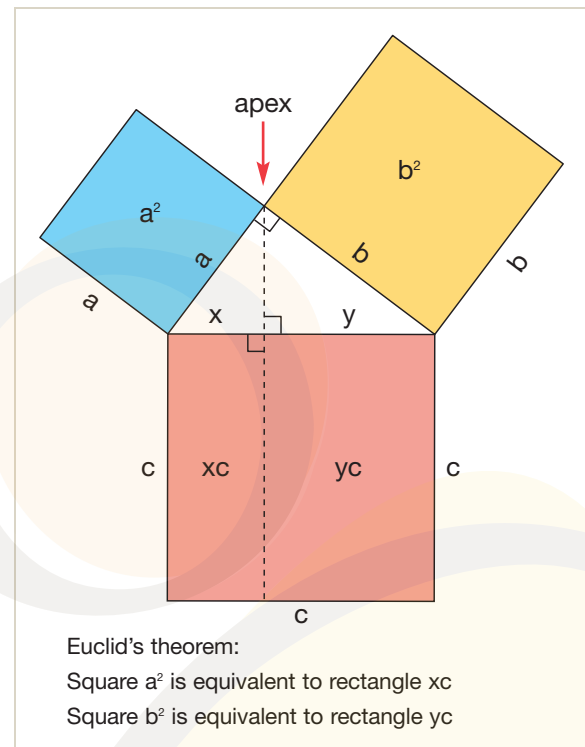
- Blue square = blue parallelogram, and blue parallelogram = small red rectangle. Therefore, blue square = small red rectangle.

In the activity about the Euclidean theorem, the students demonstrate the equivalence of the yellow square and the yellow parallelogram, and of the blue square and blue parallelogram, in two different ways:

- first, by simply substituting one shape for the other in the frame and seeing that the shapes still fill up the frame completely

- later, by using the students' knowledge of area, parallelograms, and squares to compare the squares to the parallelograms and conclude that they are equivalent

# Recognizing the 3:4:5 Right Triangle Case of the Pythagorean Theorem



Recognizing the 3:4:5 right triangle case

## Purpose

To see that the Pythagorean theorem is true by working with a 3:4:5 right triangle.
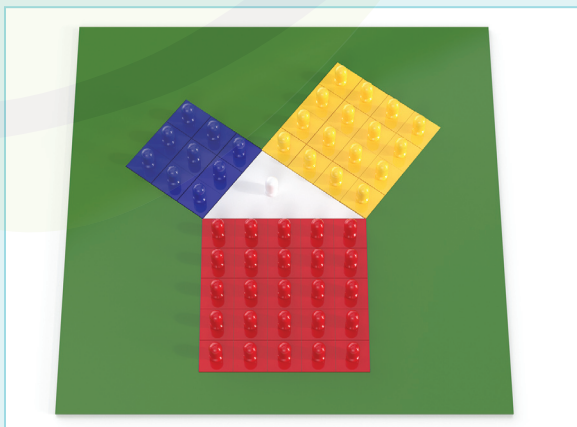
## Material

Theorem of Pythagoras material — Frame 2.

Geometry Sticks (measuring angle).

Coloring pencils.

Rulers.

Math journals and pencils.



The 3:4:5 right triangle before exchanging little squares

## Presentation

- Most Montessori teachers present this concept in Year 6.

- Students must understand the following concepts before doing this activity: area, equivalent figures, squares of numbers.

- Invite a small group of students to a mat or table to explore one way of demonstrating the Pythagorean theorem.

- Place the Theorem of Pythagoras material — Frame 2 in the work area.

## Checking the Theorem by Exchanging Little Squares

- Invite a student to pick up the white triangle and say what kind of triangle it is. (Right scalene.) Invite a student to check that it is a right triangle using the measuring angle from the Geometry Sticks. Confirm that this is a scalene triangle, with all sides of different lengths.

- Encourage the other students to handle and look at the triangle. Invite the student to replace the triangle on the tray.

- Invite a student to check that the little blue, yellow, and red squares are the same size by choosing one square of each color and holding pairs of different colors against each other, flat surfaces touching. Alternatively, the students may exchange a red square for a blue and then a yellow square on the triangle tray.

- Explain that the students are going to use the little squares to measure the sides of the triangle.

- Point to the shortest side of the triangle and ask the students how many squares long this side is. (Three.) Repeat for the other two sides.



The 3:4:5 right triangle after exchanging squares

- State that this triangle's sides are in the ratio 3:4:5.

- Invite a student to point to the square on the hypotenuse. Invite another student to point to the squares on the other two sides.

- Remind the students that according to the Pythagorean theorem, in a right triangle, the square on the hypotenuse should equal the sum of squares on the other two sides. State that if all the blue and yellow squares will fit where the red squares are, this will show the theorem is true.

- Invite the students to take turns exchanging blue squares and yellow squares for red squares until the exchange is complete.
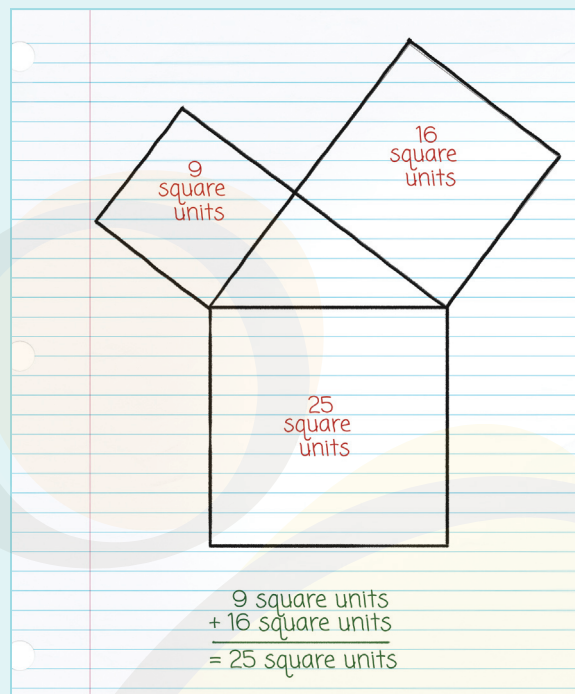


Checking the theorem by adding squares abstractly

- State that since the square on the hypotenuse is exactly filled by the squares on the other two sides, this exercise shows that the Theorem of Pythagoras is true.

## Checking the Theorem by Adding Squares Abstractly

- Explain that the students can show the Pythagorean theorem is true just by adding numbers.

- Pointing to the square on the shortest side of the triangle, invite a student to say how many little squares make up the square of three. (Nine.) Repeat for the other two sides, inviting the students to count little squares if needed.

- Ask the students to use a ruler to draw the triangle in their journals and label the sides 3 units, 4 units, and 5 units. The

students may use coloring pencils to label the sides. Ask the students to draw a square on each side. For the purposes of this illustration, the squares need not be exactly to scale. Invite the students to write "9 square units" in the smallest square, "16 square units" in the medium-size square, and "25 square units" in the largest square.

- Invite the students to test the theorem by adding 9 square units and 16 square units. Since 9 + 16 = 25, the Pythagorean theorem is again shown to be true. Ask the students to write the following statement in their journals: 9 square units + 16 square units = 25 square units.

## Extensions

- A right triangle has one side 6 units long and another side 8 units long. What is the length of the third side, the hypotenuse? Hint: The sides of the triangle must be in the proportions 3:4:5. To help solve the problem, sketch the triangle and label the sides with their lengths.

- Trace and cut out all the geometric shapes in the Theorem of Pythagoras material — Tray 2. Use the cut-out shapes to make a poster explaining Pythagoras' theorem in words and pictures.