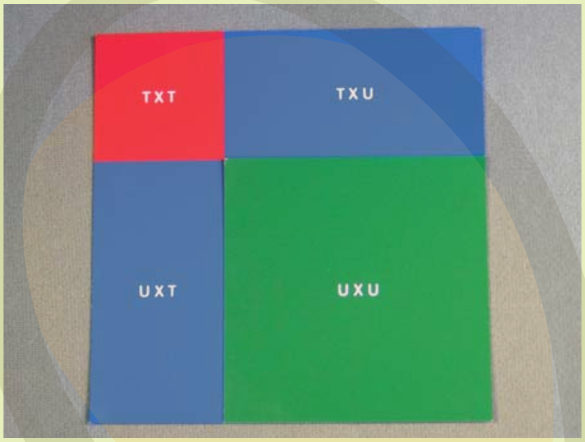
ACTIVITY 4

Finding the Square Root of a Number with Two Periods



The Binomial Guide Square layered with the Monomial Guide Square

Purpose

To understand how to find the square root of a number with two periods.

Material

Table of squares (on the curriculum support material CD for Negative Numbers and Pre-Algebra).

Illustration titled Place values and their colors (on the curriculum support material CD for Negative Numbers and Pre-Algebra).

Algebraic Pegboard Material.

Binomial and Monomial Guide Squares.

Summary sheet, Three steps involved in finding the square root of a number with two periods on the Algebraic Pegboard.

Graph paper.

Slips of white paper.

Felt markers.

Whiteboard and whiteboard markers.

Math journals and pencils.

Presentation

- Most Montessori teachers present this concept in Year 6.
- Invite a small group of students to a mat or table to learn how to find the square root of a number with two periods.

REVIEWING CONCEPTS AND SETTING UP THE MATERIAL

- Review with the students that they have learned to find the square roots of numbers with one period. Explain that the students will now work on finding square roots of numbers with two periods using the Algebraic Pegboard Material. Invite the students to say what range of numbers this suggests. Discuss and confirm that the smallest whole number with two periods is 100 (1'00) and the largest is 9,999 (99'99).
- Invite the students to say how many digits the square root will have if the radicand lies between 100 and 9,999. Discuss and clarify that the square root will have two digits because the radicand has two periods. Explain that when finding the square root of a number with two periods, the square root can be expressed as a binomial because it will have two digits: a 10s value and a unit. For example, $\sqrt{6'25} = 25$, which can be expressed as the binomial (20 + 5).

- Present the illustration titled Place values and their colors, and review with the students the values of the pegs up to 100,000. Then present the Table of squares and explain that the value 625 will be used as the example in this presentation. Discuss and clarify that 625 has a square root of 25, since 25 x 25 = 625.

- Invite a student to write the radicand 625 under the radical sign, and then to divide the value into periods: √6'25. With the students, discuss and clarify that because the radicand has two periods, the square root will have two digits. This means there will be a 10s value and a unit value in the square root. Invite a student to record these on the whiteboard as follows: √6'25|TU.

- Point out that this means the square root of 625 can be expressed in the form of the binomial (20 + 5). Explain that because the square root of 625 can be presented as a binomial (20 + 5), the square of that binomial square root can also be presented as the square of the binomial: $(20 + 5)^2$.

- Demonstrate the Binomial Guide Square and remind the students that the values that are multiplied together to produce the binomial square can be found along the bottom and right-hand sides of the square. This means that if the square represents a number, the sides represent the square root of the number.

- Also remind the students that the Binomial Guide Square presents the arrangement of hierarchical values for the square of a binomial. Review how the 10s and units across the bottom and right sides multiply to produce the interior values of the square.

- Invite the students to choose the appropriate pegs to represent 625 on the Algebraic Pegboard. Confirm that the value is too large to be made from units, and that it is necessary to also use 10s and 100s. Invite a student to express 625 on the whiteboard as a hierarchical trinomial consisting of 100s (H), 10s (T), and units (U) (600 + 20 + 5). Invite another student to place the appropriate numbers of pegs in the appropriately colored cups (6 x H, 2 x T, 5 x U).

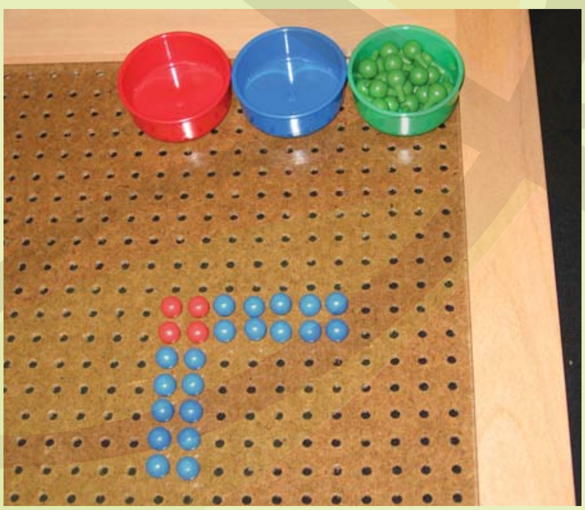
Preparing the Algebraic Pegboard to find the square root of 625

FINDING THE SQUARE ROOT OF 625 ON THE ALGEBRAIC PEGBOARD

- Invite the students to begin building a square of 625 to see how many complete squares can be built, beginning with the 100s. Using the Binomial Guide Square as a guide, the students begin by placing 1 x H peg. Confirm that the students understand that this is equal to 100. Next, ask the students to add 3 x H pegs around the 1 x H peg, creating a 4 x H square. Confirm that the students understand this square is equal to 400.

Building the largest possible square with 100s pegs, 4 X H square

- Invite the students to try to add another row to the bottom and right sides with the 100s pegs. (They will not be able to.) Explain that the students need to exchange the remaining 100s pegs for 10s, giving 20 more 10s, for a total of 22.



Building the largest possible rectangles with 10s pegs

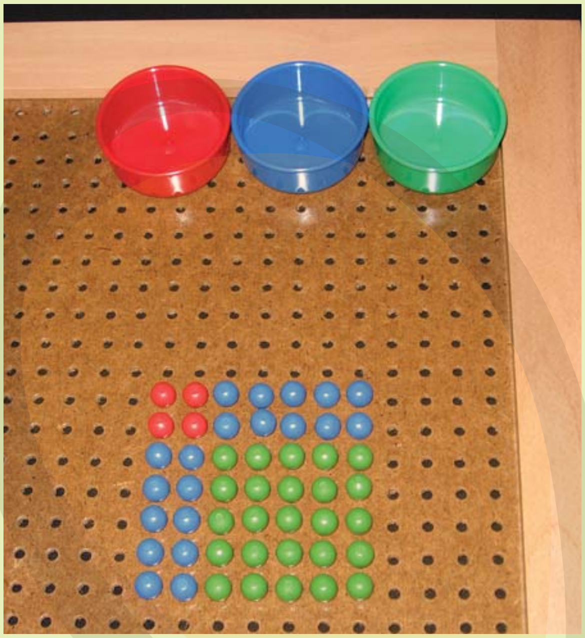
- Again, point to the Binomial Guide Square and invite the students to say where the 10s pegs will be placed. Confirm that the pegs will form two equal rectangles, one below and one to the right of the existing square of 100s pegs. Invite the students to build the largest possible rectangles, keeping both of equal size. The students will start by adding two pegs to the bottom and right sides of the 100s square, and then repeat this step four more times, until they have two remaining pegs. Point out that the students cannot add the remaining two 10s pegs, and so must exchange the last two 10s pegs for units. This gives 20 more unit pegs, for a total of 25 unit pegs.

- Invite the students to look at the layered Binomial and Monomial Guide Squares and determine where the unit pegs go. Confirm that the students will fill in the remaining square. Add that the unit pegs will fill the square exactly only if the 100s and 10s have been done correctly. Invite the students to place the unit pegs, which fill the remaining space completely with no remainder.

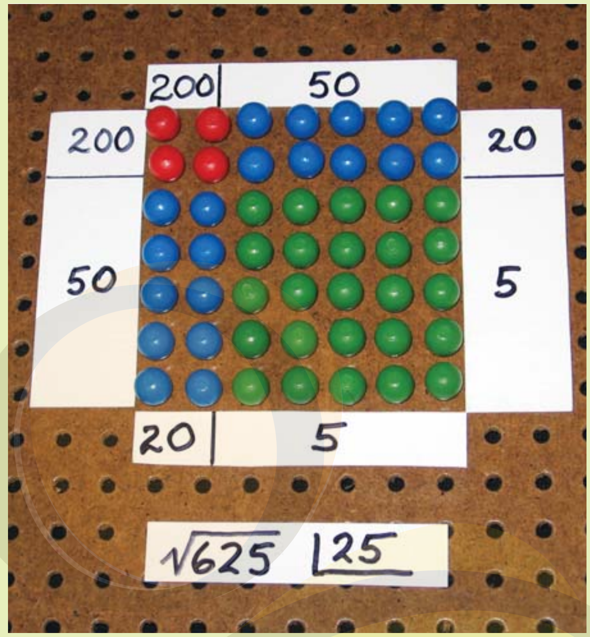
- Point out to the students that they have built a complete binomial square. Then invite the students to find the square root of 625 using the square. Confirm that the students find the square root by counting the values of the pegs along the bottom or right side. Each of these sides has two 10s and five units. Review that these values can be written as a binomial (20 + 5) or as a monomial (25).

- Ask the students to express the square root of 625 in their journals using the radical sign: $\sqrt{625}|25$.

- Encourage the students to examine the process by which they arrived at the square root. In particular, encourage the students to notice how the individual



Completing the square of 625 with units pegs



The square root of 625 is 25

digits of the square root could be found as the binomial square took shape. For example, the initial square of 100s pegs had two pegs on each side, and this gave the first digit of the square root because it also determined the width of the rectangles of 10s pegs. Then point out that the length of the rectangles of 10s pegs determined the dimensions of the units square, which gave the second digit of the square root.

REVIEWING THE STEPS IN FINDING THE SQUARE ROOT

- Review that there were three steps to finding the square root of a number with two periods. Add that these steps also apply to finding the square root abstractly. Summarize the steps as follows:

  Step 1 is to find the first digit of the square root. This is found by building the square of 100s pegs, because the length of that square's sides determines the width of the rectangles of tens. Add that in this step, the students are finding the largest number with a square that will fit into the 100s. In this example, the largest number was 20, giving a square of 400. The remaining 100s are then exchanged for tens.

  Step 2 is to find the second digit by building the largest possible rectangles of 10s pegs. The length of the rectangles equals the second digit — in this example, 5. This is done by building up the two rectangles equally. The number of pegs added each time will be equal to two times the value of the first digit, since the width of the rectangles is equal to the width of the 100s square. In this example, this meant adding four 10s at a time, two to each rectangle. Any remaining 10s are then exchanged for units.

Step 3 is to confirm the second digit by completing the square with units. If the units fit exactly, then the square root is correct and there is no remainder. If there are not enough units to complete the square, then the second digit chosen is too large and must be reduced until the units do complete the square. If there are units left over, then they give the value of the remainder.

- Demonstrate the summary sheet, Three steps involved in finding the square root of a number with two periods on the Algebraic Pegboard, and encourage the students to refer to the sheet as needed when working further with square roots of numbers with two periods.

- Ask the students to use their journals to summarize the process they followed in this activity, using graph paper to illustrate the steps involved.

Extension

- Repeat this activity with several numbers between 100 and 9,999 that have square roots without remainders, and then advance to numbers with square roots with remainders.

