

MULTIPLYING AND DIVIDING FRACTIONS

Background Information

In lower elementary, most students will have learned to multiply and divide fractions by whole numbers, using the Fraction Circles. In upper elementary, students review those basic skills and then advance to multiplying and dividing various combinations of fractions and whole numbers. For example, students learn to multiply fractions by fractions and divide whole numbers by fractions.

Multiplying fractions

Just as with whole numbers, fractions to be multiplied may be called multiplicands and multipliers. The result of multiplying fractions is called the product.

To multiply a fraction by a whole number, the numerator is simply multiplied by the whole number. For example, the sentence “Two quarters equal one half” can be written as the following equation: $2 \times \frac{1}{4} = \frac{1}{2}$. In this equation, the numerator 1 is multiplied by the whole number 2 to give the intermediate product $\frac{2}{4}$, which equals $\frac{1}{2}$. In another example, to multiply $\frac{4}{17}$ by 3, the numerator 4 is multiplied by the whole number 3, for a product of $\frac{12}{17}$.

To multiply fractions by fractions, the numerators are multiplied together and the denominators are multiplied together. For example, the sentence “One-half of one-half equals one-quarter” can be written as the following equation: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The multiplication sign acts like the word “of.” In another example, $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$.

Dividing fractions

Just as with whole numbers, a fraction to be divided is called the dividend, and the

result is called the quotient. The number by which the dividend is being divided is called the divisor. Montessori students initially learn to divide fractions by sharing out fraction pieces among skittles that represent the divisor, just as they did when dividing whole numbers.

To divide a fraction by a whole number, the numerator is simply divided by the whole number. For example, to divide $6/15$ by 3, the numerator 6 is divided by the whole number 3, for a quotient of $2/15$.

When dividing fractions using the material, individual pieces of the material cannot be cut up. As a result, it is sometimes necessary to express a fraction in different terms so the divisor will divide evenly into the numerator. This is done by finding a common multiple for the numerator and the divisor. For example, the equation $3/5 \div 2$ can be expressed as $6/10 \div 2$. Two divides evenly into 6, giving a quotient of $3/10$ for the equation. Students apply this concept by working with Fraction Circles and experimenting until they find an equivalent fraction that can be divided evenly.

To divide a whole number by a fraction, students use a fraction skittle to represent the divisor. For example, to divide 2 by $3/4$, students divide two whole circles among three one-fourth Fraction Skittles. The steps are as follows:

- Set up three one-fourth Fraction Skittle pieces.
- Divide the two whole Fraction Circles among the three Fraction Skittle pieces. Since 2 will not divide evenly among three pieces, the Fraction Circles must be

made into equivalent fractions that will divide evenly into 3. Using trial and error, the students divide each whole circle into thirds. It is now possible to place 2 thirds on each Fraction Skittle piece.

- Just as with whole numbers, the quotient is always the amount one whole in the divisor gets. Since each one-fourth of the divisor gets $2/3$, then the whole divisor gets $4 \times 2/3$, or $8/3$. The quotient is $8/3$.

Dividing a fraction by a fraction is done the same way as dividing a whole number by a fraction. That is, a fraction skittle represents the divisor, and the steps are the same. The only difference is that instead of sharing out a whole number among the Fraction Skittle piece(s), the student must share out a fraction.

Simplifying fractions

Some equations have answers in which the numerator is larger than the denominator — for example, $12/6$ or $5/2$. It is often desirable to simplify such fractions — in other words, express each fraction as an equivalent whole number or as a whole number plus a fractional part. Simplifying fractions is addressed in the section of this manual titled Improper Fractions and Mixed Numbers. Teachers wanting to introduce the concept of simplifying fractions may choose to present Activity 1 from that section at any time.

ACTIVITY 1

Multiplying Fractions by Whole Numbers, Using the Fraction Circle Box

Purpose

To practice multiplying fractions by whole numbers, and learn to multiply fractions greater than 1 by whole numbers.

Material

Fraction Circle Box.

Prepared equation slips for multiplying fractions by whole numbers (on the curriculum support material CD for Whole Numbers and Fractions for fractions equal to 1 or less; also, the teacher should prepare some equation slips with fractions greater than 1, such as $12/7 \times 2$).

Whiteboard and whiteboard markers.

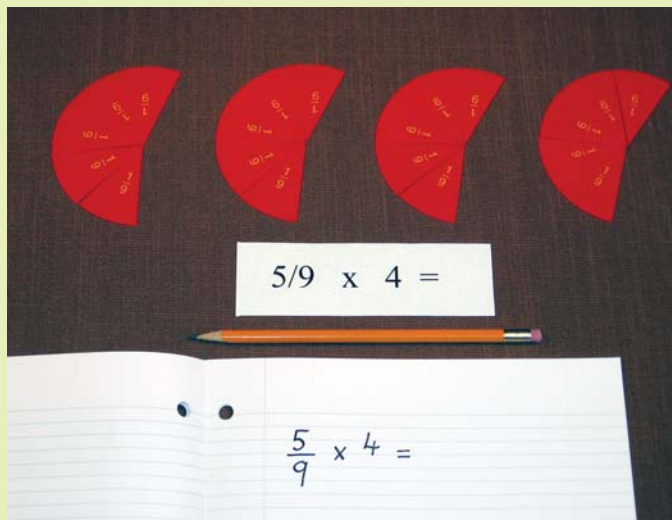
Math journals and pencils.

Presentation

- Most Montessori teachers review this concept in Year 4 and again in Year 5.
- Most students will be familiar with multiplying fractions equal to 1 or less by whole numbers. The Fraction Circle Box enables them to practice multiplying with fractions greater than 1.

SETTING UP THE EQUATION

- Invite a student to get the Fraction Circle Box and bring it to a table or mat to learn to multiply fractions.



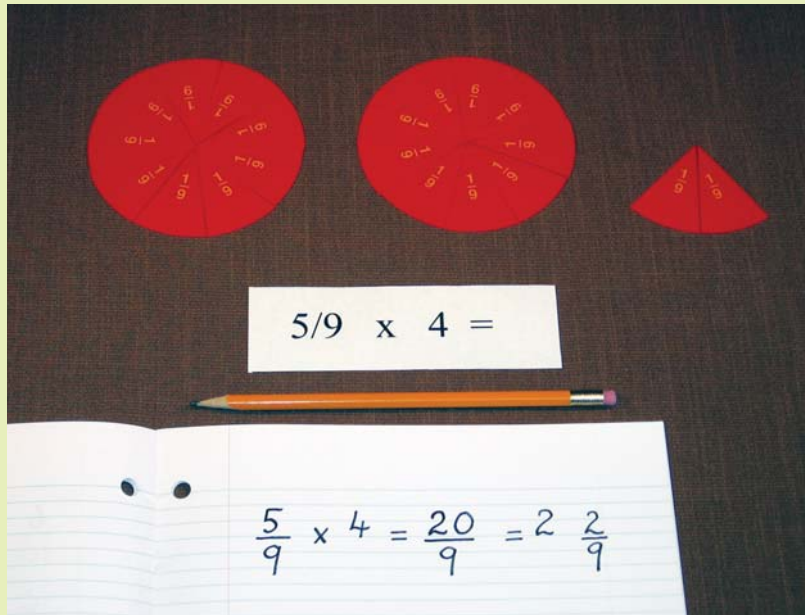
Setting out 5/9, four times

- Invite the student to place the Fraction Circle Box at the top of the work area.
- Encourage the student to choose an equation slip and read it — for example, $5/9 \times 4$.
- Tell the student that this equation means taking five ninths, four times.
- Write the equation on the whiteboard in the format illustrated.
- Ask the student to write the equation $5/9 \times 4 =$ in her/his journal in the format illustrated.
- Encourage the student to take 5 ninths fraction pieces and arrange them in the work area to form part of a circle.

- Invite the student to repeat this three more times, leaving a space between each group of 5 ninths. He/she will end up with four groups of $\frac{5}{9}$.

SOLVING THE EQUATION

- Invite the student to count the fraction pieces in the first group and say how many there are. (Five.)
- Invite the student to say how many fraction pieces are in the other three groups. (Five per group.)
- Encourage the student to say how many fraction pieces there are in all. (Twenty.)
- State that $\frac{5}{9}$ taken 4 times equals $\frac{20}{9}$. In other words, $\frac{5}{9} \times 4 = \frac{20}{9}$.
- Ask the student to complete the equation in her/his journal: $\frac{5}{9} \times 4 = \frac{20}{9}$. Write the equation on the whiteboard.
- Encourage the student to rearrange the fraction pieces to form as many circles as possible. The student will make 2 circles with 2 ninths fraction pieces left over.
- Invite the student to say the simplified fraction aloud: $2 \frac{2}{9}$. Encourage the student to record the fraction $2 \frac{2}{9}$ in his/her journals.
- Encourage the student to place the fraction pieces back in the Fraction Circle Box.



Combining all the fraction pieces to find the total, $\frac{20}{9}$

- Encourage the student to repeat with several more prepared equations until she/he is proficient at multiplying fractions by whole numbers.

Extensions

- Copy a prepared equation into the journal and try to find the answer without using the Fraction Circles or Fraction Circle Box. Repeat for other prepared equations.
- A student is using Imperial cup measures to prepare a triple batch of a pasta dish for a party. The original recipe calls for $\frac{1}{3}$ cup black olives, $\frac{2}{3}$ cup feta cheese, and $\frac{1}{2}$ cup fresh parsley. What quantities does the student need after tripling the recipe?
- Open any recipe book and practice doubling, tripling, or quadrupling two or three recipes.

ACTIVITY 2

Multiplying Fractions by Fractions, Using Fraction Circles and the Fraction Circle Box

Purpose

To learn how to multiply fractions by fractions.

Material

Fraction Circles.

Fraction Circle Box.

Equation slips for multiplying fractions by fractions, prepared by the teacher.

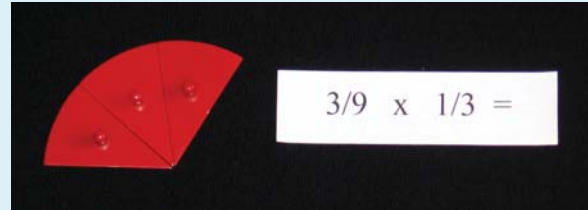
Math journals and pencils.

Presentation

- Most Montessori teachers present this concept in Year 4 and review it in Years 5 and 6.

SOLVING AN EASY FIRST EQUATION

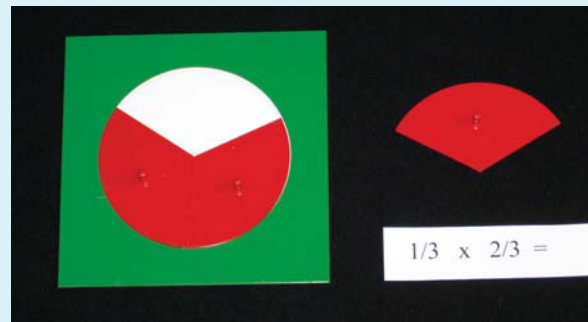
- Invite a student to get the Fraction Circles and bring them to a table or mat to learn to multiply fractions by fractions.
- Invite the student to place the Fraction Circles at the top of the work area.
- Encourage the student to choose an equation slip and read it — for example, $3/9 \times 1/3$.
- Tell her/him that this equation means taking $1/3$ of $3/9$.
- Ask the student to write the equation in his/her journal as follows: $3/9 \times 1/3 =$.



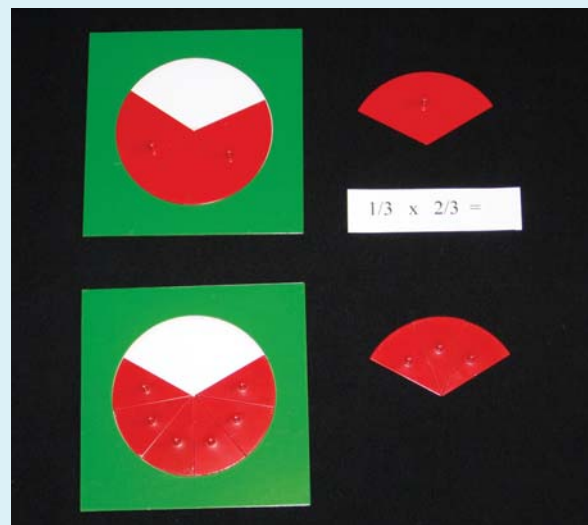
Setting up $3/9 \times 1/3$



Finding that $3/9 \times 1/3 = 1/9$



Setting up $1/3 \times 2/3$



Finding that $1/3 \times 2/3$ is equivalent to $3/9 \times 2/3$



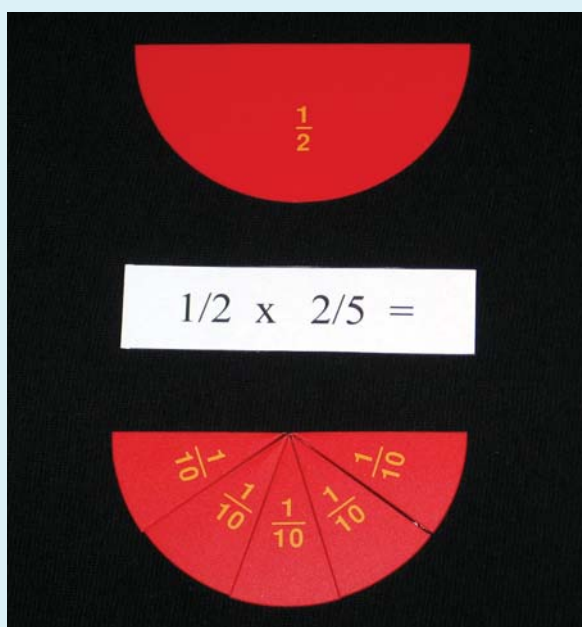
Finding that $1/3 \times 2/3 = 2/9$

- Encourage the student to make the fraction $3/9$ in the work area, using fraction pieces.
 - Invite the student to divide $3/9$ into thirds.
 - The student will divide the fraction into three parts. Each part will contain $1/3$ of $3/9$, which equals $1/9$.
 - State that $3/9 \times 1/3 = 1/9$.
 - Ask the student to record the answer in her/his journal.
 - Repeat with other easy equations such as $4/7 \times 1/2$, $2/3 \times 1/2$, and $3/6 \times 1/3$.
- SOLVING A MORE ADVANCED EQUATION, WHERE AN EQUIVALENT FRACTION MUST BE MADE**
- The same day or a different day, still working with the Fraction Circles, invite the student to choose an equation slip with a fraction that needs to be written as an equivalent fraction before it can be split up, such as $1/3 \times 2/3$.
 - Remind the student that $1/3 \times 2/3$ means $2/3$ of $1/3$.
 - Encourage the student to make the fraction $1/3$ in the work area, using Fraction Circles.
 - Ask the student if he/she can take $2/3$ of the $1/3$ fraction piece. (No.)
 - Encourage the student to make a fraction that is equivalent to $1/3$ and that can be divided into three. Using trial and error, the student may first try $2/6$ before realizing that $3/9$ will work.
 - Invite the student to write this equivalent equation in her/his journal: $3/9 \times 2/3 = .$
 - Invite the student to take $2/3$ of $3/9$ and place the pieces in the work area.
 - Encourage the student to count the ninths fraction pieces. There are two.
 - Say, “ $3/9 \times 2/3 = 2/9$.”
 - Ask the student to complete this equation in his/her journal.
 - Ask the student to place the fraction pieces back in the Fraction Circle frames at the top of the work area.

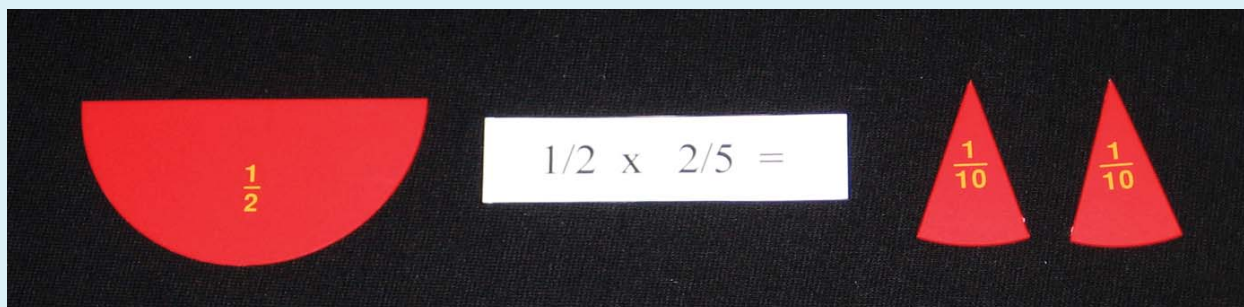
- Repeat with other equations where an equivalent fraction must be made, such as $1/4 \times 1/2$ or $1/3 \times 1/3$.

YEARS 5 AND 6

- Repeat the above activities with new numbers, only this time use the Fraction Circle Box instead of or in addition to the Fraction Circles. Use more challenging fractions such as $1/2 \times 2/5$ or $2/3 \times 4/2$.



Setting up $1/2 \times 2/5$ and making a fraction equivalent to $1/2$



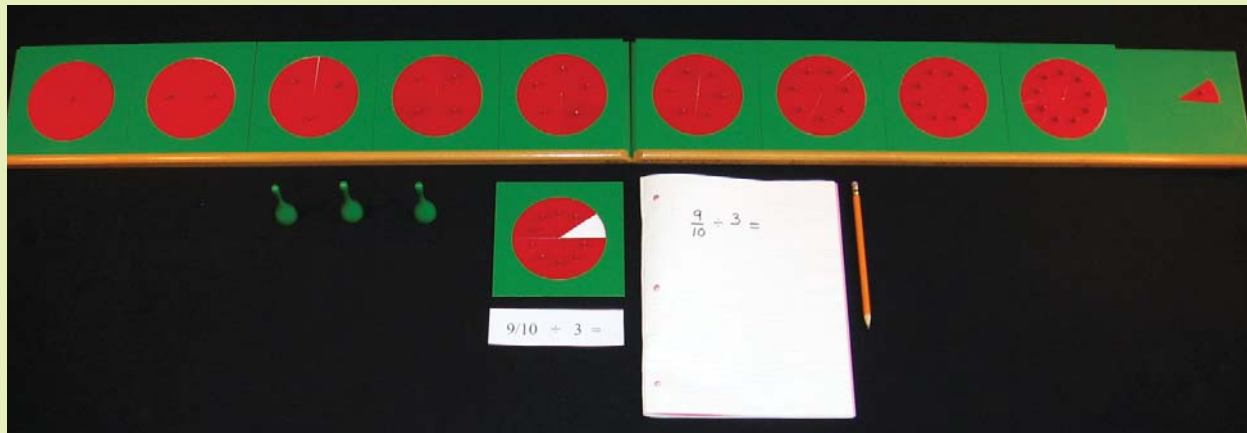
Finding that $1/2 \times 2/5 = 2/10$

Extensions

- Copy a prepared equation into the journal and try to find the answer without using the Fraction Circles or Fraction Circle Box. If necessary, make little sketches to represent the fractions. Repeat for other prepared equations.
- A single adult wants to make one-half a recipe of berry sauce for ice cream using Imperial cup measures. The original recipe calls for 3 cups fruit, 2 tsp cornstarch, $1/2$ cup sugar, and $1/4$ cup water. Write the four equations that must be solved to halve the recipe, and find the answers. Record the one-half recipe in the journal.
- Use Fraction Circles to solve the following word problem, and write the equation in the journal: The class was discussing where to go on the next field trip. One-half wanted to go to the aquarium. Of the one-half who wanted to go to the aquarium, two-thirds wanted to watch the sharks being fed. What fraction of the whole class was interested in watching sharks being fed?

ACTIVITY 3

Dividing Fractions by Whole Numbers, Using Fraction Circles and the Fraction Circle Box



Setting up $9/10 \div 3$

Purpose

To learn how to divide fractions by whole numbers.

Material

Fraction Circles.

Fraction Circle Box.

Large Green Skittles.

Prepared equation slips for dividing fractions by whole numbers (on the curriculum support CD for Whole Numbers and Fractions: also, the teacher should prepare some equation slips with fractions greater than 1, such as $12/5 \div 4 =$).

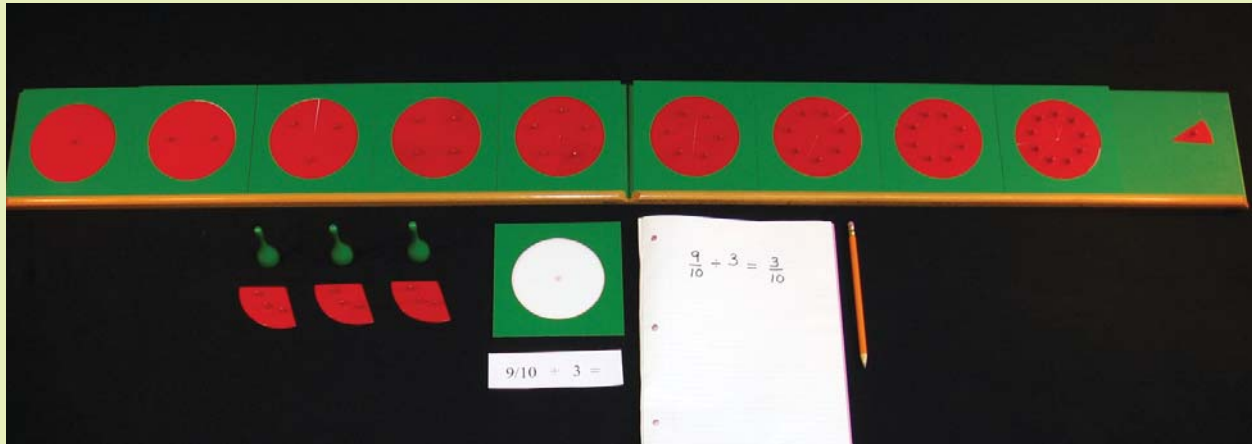
Math journals and pencils.

Presentation

- Most Montessori teachers review this concept using the Fraction Circles in Year 4 and the Fraction Circle Box in Year 5.

SETTING UP THE EQUATION

- Invite a student to come to a table or mat where the material is set out to learn how to divide fractions by whole numbers.
- Invite the student to lay out the Fraction Circles in order.
- Invite the student to choose an equation slip and read it — for example, $9/10 \div 3 =$.
- Ask the student to copy the equation in his/her journal .
- Review the terms dividend and divisor with the student.
- Invite the student to place the ninths Fraction Circle frame in the center of the work area.
- Encourage the student to make the dividend in the frame by leaving nine of the tenths fraction pieces in the frame and place the extra one on the board.



Finding that $9/10 \div 3 = 3/10$

SOLVING THE EQUATION

- Ask the student to select enough Large Green Skittles to make the divisor and place them in the work area. For the example $9/10 \div 3$, use three skittles.
- Remind the student that division means sharing equally.
- Encourage the student to begin sharing equally, placing one fraction piece under each skittle until they are all shared.
- Remind the student that the amount one skittle gets is the answer to the division question.
- Ask the student what the answer is to $9/10 \div 3$. (It is $3/10$.)
- Ask the student to write the answer in her/his journal.
- Encourage the student to place the tenths fraction pieces back in their Fraction Circle frame and put the tenths Fraction Circle back on the board.
- Encourage the student to continue in the same manner with another question.

YEAR 5: FRACTION CIRCLE BOX

- Once students are used to dividing fractions using the Fraction Circles, have them do the above activity using the Fraction Circle Box instead.
- The only difference is that instead of making the dividend using Fraction Circles, the student will make the dividend using fraction pieces from the Fraction Circle Box.
- When students are familiar with the material, introduce them to dividing fractions larger than 1. For example, they could divide $24/3$ by 2, $12/5$ by 6, or $30/8$ by 10.

Extensions

- Select a prepared equation slip and find the answer without using the material. Repeat for three or four other prepared equations.
- A store manager has $8/9$ of a truckload of boxes to unload, and two employees to share the job. What fraction of a truckload must each employee unload?

ACTIVITY 4

Dividing Fractions by Whole Numbers When Equivalent Fractions Must Be Made First

Purpose

To learn how to divide fractions by whole numbers when equivalent fractions must be made first.

Material

Fraction Circles.

Fraction Circle Box.

Large Green Skittles.

Prepared equation slips (on the curriculum support material CD for Whole Numbers and Fractions for fractions equal to 1 or less; also, the teacher should prepare some equation slips with fractions greater than 1 for the Year 5 work).

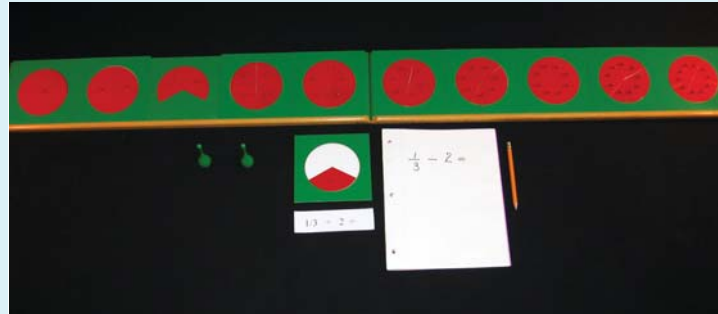
Math journals and pencils.

Presentation

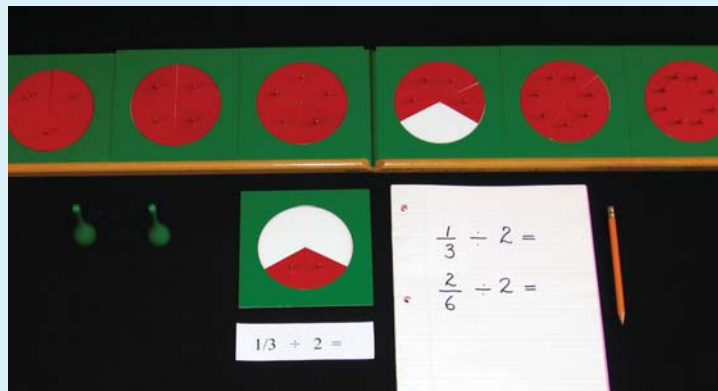
- Most Montessori teachers present this concept in Year 5 and review it in Year 6.
- Invite a student to come to a table or mat where the material is set out. Tell the student she/he will learn more about dividing fractions by whole numbers.

SETTING UP THE EQUATION

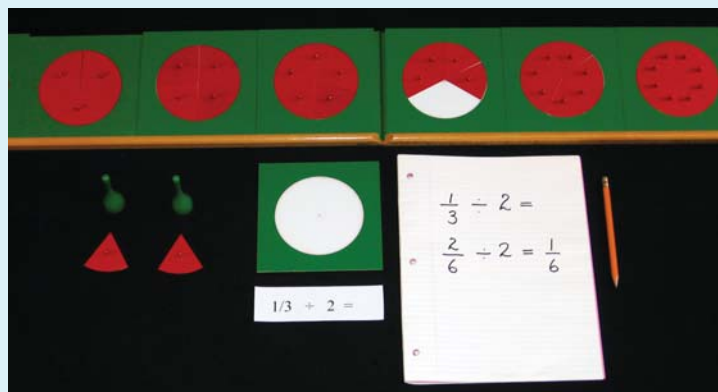
- Invite the student to choose an equation slip and read it — for example, $1/3 \div 2 = .$



Setting up the equation $1/3 \div 2$



The equivalent fraction $2/6$ is divisible by 2



The quotient is $1/6$

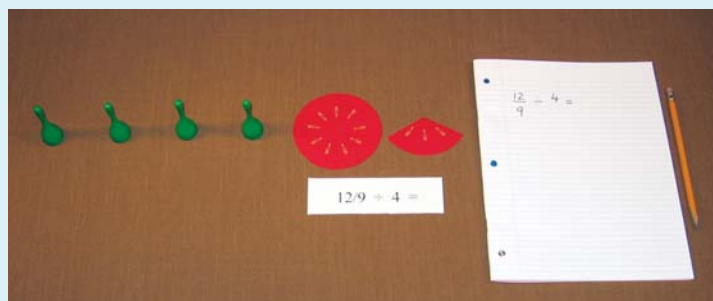
- Ask the student to record the equation in his/her journal.
- Encourage the student to place the thirds Fraction Circle frame in the center of the work area.

- Invite the student to make the dividend $1/3$ in the frame by leaving one of the thirds in the frame and placing the other two back on the board.
- Invite the student to select the right number of Green Skittles to make the divisor and place them to the left of the frame. (Two skittles, for this example.)
- Ask the student what difficulty there will be in dividing. (One fraction piece cannot be shared equally among two skittles.)

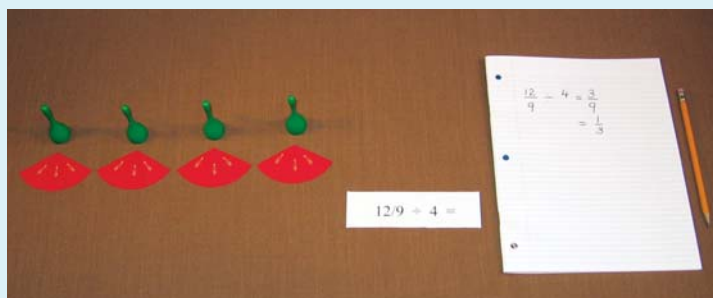
SOLVING THE EQUATION

- Encourage the student to find a fraction equivalent to $1/3$ that can be shared equally between two skittles. Hint: The numerator needs to be divisible by 2.
- Once the student has found an equivalent fraction that can be shared equally between two skittles, ask her/him to record the equivalent equation in the journal as follows: $2/6 \div 2 = .$
- Invite the student to remove the thirds fraction piece from the Fraction Circle frame and instead place two sixths fraction pieces in the Fraction Circle frame.
- Encourage the student to share fraction pieces equally between the two skittles until the fraction pieces have been used up.
- Ask the student how much each skittle gets. (One sixth.)

- Ask the student to complete the equation in his/her journal as follows:
 $2/6 \div 2 = 1/6.$
- Ask the student to complete the original equation as well: $1/3 \div 2 = 1/6.$
- Encourage the student to put the sixths fraction pieces back in their Fraction Circle frame on the Fraction Circle board.
- Encourage the student to continue in the same manner with more prepared equation slips until she/he is proficient at dividing fractions that require finding equivalent fractions.



Setting up the equation $12/9 \div 4$



The quotient of $12/9 \div 4$ is $3/9$, or $1/3$

YEAR 5: FRACTION CIRCLE BOX

- Once students are used to dividing fractions using the Fraction Circles, have them do the above activity with different equations, using the Fraction Circle Box. This material can handle fractions with

numerators greater than one, such as $12/9$ or $15/4$.

- The only difference is that instead of making the dividend using Fraction Circles, the student will make the dividend using fraction pieces from the Fraction Circle Box.
- Encourage students to reduce the quotients where possible.

Extensions

- The teacher has one-half box of tissues to share among four students with colds. How much tissue will each student get? How much will each student get if the teacher has only one-third box?
- There are four quarters in a dollar. Suppose there was a coin worth one-half a quarter. What fraction of a dollar would that coin be worth?

ACTIVITY 5

Dividing Whole Numbers by Fractions, Using the Fraction Circle Box

Purpose

To learn how to divide whole numbers by fractions, using the Fraction Circle Box.

Material

Fraction Circle Box.

Fraction Skittles.

Equation slips for dividing whole numbers by fractions, prepared by the teacher.

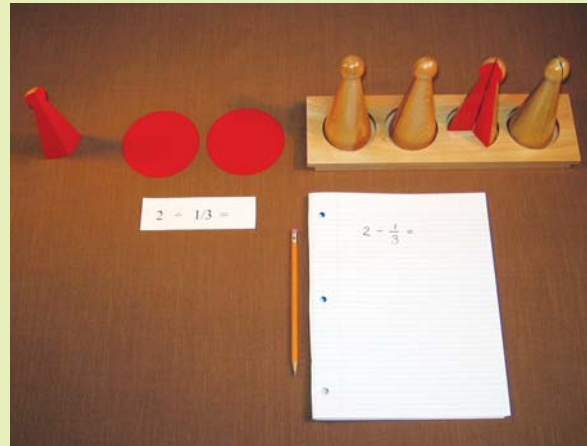
Math journals and pencils.

Presentation

- Most Montessori teachers present this concept in Year 5 and review it in Year 6.
- Invite a student to a table or mat where the material is set out. Tell the student he/she will learn to divide whole numbers by fractions.

SETTING UP THE EQUATION

- Invite the student to choose an equation slip and read it — for example, $2 \div \frac{1}{3} =$.
- Ask the student to record the equation in her/his journal.
- Invite the student to place two whole fraction pieces in the center of the work area to represent the 2.
- Invite the student to choose the Fraction Skittle that is divided into thirds.

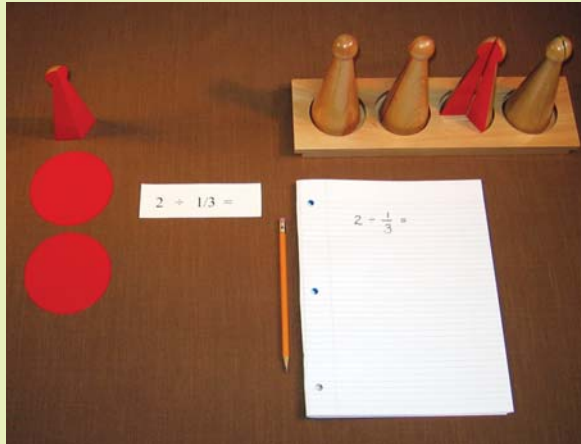


Setting up the equation $2 \div \frac{1}{3} =$

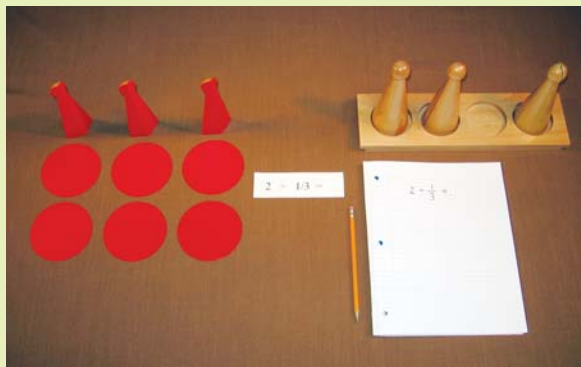
- State that since the division is by one-third, the student needs only one of the one-third Fraction Skittle pieces. Invite the student to place one Fraction Skittle piece in the work area to the right of the fraction pieces.

SOLVING THE EQUATION

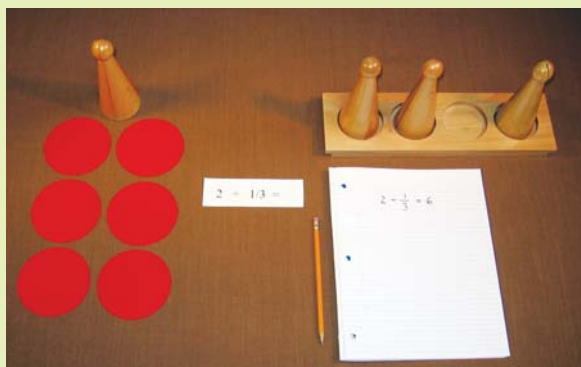
- Invite the student to share the two whole fraction circles out to the thirds Fraction Skittle piece.
- Since there is only one Fraction Skittle piece, both whole circles are placed beside the thirds Fraction Skittle piece.
- Remind the student that in division, the answer to the equation is always the amount that a whole skittle gets.
- Invite the student to place the other two pieces of the Fraction Skittle in the work area.



The Fraction Skittle piece gets 2 whole circles



The two other Fraction Skittle pieces also get 2 whole circles each



The quotient is 6

- State that each third gets two whole circles.
- Encourage the student to place two whole circles beside each thirds Fraction Skittle piece.
- Invite the student to push the Fraction Skittle pieces together to make a whole Fraction Skittle.
- Invite the student to push all the whole fraction pieces together and count them. (Six.)
- Ask the student how many the whole Fraction Skittle got. (Six.) State that this is the answer to the division equation.
- Encourage the student to record the answer in his/her journal as follows: $2 \div \frac{1}{3} = 6$.
- Encourage the student to repeat with more prepared equation slips until she/he is proficient at dividing whole numbers by fractions.

Extension

- Choose six prepared equations that have a 1 in the numerator. Solve the equations using the material. Look carefully at all the equations' answers to see if there is a pattern. How can a person divide a whole number by a fraction without using the material? Write the answer in the journal.